A geometric approach to differential-algebraic systems

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Chapter 8

Conclusion and Future Works

In this thesis, we focused on the following two parts. In the first part, we considered the analysis of differential-algebraic (DAE) systems. In particular, we studied the notion of bisimulation relation of DAE systems. Also we studied the disturbance decoupling problems for a particular class of DAE systems. In the second part, we investigated the problem of control by interconnection based on abstraction system.

The contributions of the thesis are summarized in Section 8.1. The recommendations for future research are given in Section 8.2.

8.1 Contributions

In Chapter 3 we studied by methods from geometric control theory to characterized the solution set of differential-algebraic (DAE) systems. We restricted ourselves to continuous and piecewise-differentiable solutions corresponding to consistent initial conditions. We extend the standard definition of consistent subset for the case of DAE systems with arbitrary input functions and disturbance functions modelling internal non-determinism. Next, we used geometric control theory in order to explicitly describe the set of consistent states and the set of state trajectories.

Based on the results in the previous chapter, in Chapter 4 we defined and studied by methods from geometric control theory the notion of bisimulation relation for general linear differential-algebraic systems, including the special case of DAE systems with regular matrix pencil. We determined the linear-algebraic conditions for the existence of a bisimulation relation, directly in terms of the differential-algebraic equations instead of computing the solution trajectories. We also studied the one-sided notion of bisimulation relation called simulation relation. Lastly, we discussed the abstraction of DAE systems.

In Chapter 5 we continued the development of the notion of bisimulation relation for DAE systems. We studied a different notion of bisimulation relation for DAE systems. In this chapter, we restricted ourselves to DAE systems with regular matrix pencil $(sE - A)$. We also took into account the state outside the consistent subset. The regularity assumption is equivalent to the fact that the systems can be decoupled into two subsystems, namely the slow subsystem and the fast subsystem. It also guarantees that the solution of regular DAE systems is the direct sum of the solution of the slow subsystem and the fast subsystem. Based on that idea, the notion of bisimulation relation for regular matrix pencil DAE systems was constructed by computing the notion of partial bisimulation relation related to slow subsystem and fast subsystem. We also provided the notion of simulation relation for regular DAE systems. This simulation relation is constructed by computing the simulation relation between the slow subsystem and the fast subsystem. Furthermore, the abstraction for regular DAE systems in this case was given as the abstraction for each subsystem.

In Chapter 6 we studied the disturbance decoupling problem (DDP) for linear systems with complementarity switching. These systems are a particular class of switched (or, hybrid) DAE systems. We restricted attention to switching behaviors of the system satisfying the extra condition that the state vector at the switching instant belongs to the consistent subset of the new mode. First we began to derive necessary and sufficient geometric conditions such that the DAE system is disturbance decoupled. Next, we extended the result to linear systems with complementarity switching. We formulated a necessary condition and a sufficient condition such that the linear system with complementarity switching is disturbance decoupled. In general, these two conditions do not coincide.

In Chapter 7 we considered the control by interconnection problem based on abstraction. In the first problem, we considered a control by interconnection based on abstraction system. We defined necessary and sufficient conditions for the existence of a controller for abstraction system such that the abstraction system interconnected with the controller system is bisimilar to a given specification system. Next we considered the problem of applying the controller system derived for the abstraction system to the original plant system. Here we made a distinction between the situation where the set of control variables of the abstraction system is equal to the set of control variables of the plant system, and the more general situation where this is not anymore the case. In this last case we needed to modify the interconnection of the controller to the original plant system. The main theorem consisted of showing that the resulting interconnection of the original plant system and the controller system derived for the abstraction system is *simulated* by the specification system. We showed that for linear abstraction systems the problem of application of the controller constructed for the abstraction system to the original plant system admits a direct and elegant solution within the framework of geometric control theory. Finally, for an initial study on feedback controller, we showed that if there exists a feedback controller for abstraction system achieving the specification system then this controller can be applied to the original plant system such that the closed-loop system is simulated by the specification system.

8.2 Recommendations for future work

Some possible research directions are the following.

In Chapter 4 a possible question is how to reduce a linear DAE system to a system with lower state space dimension and minimal number of state equations which is bisimilar to the original system. In particular, how to reduce the DAE systems to a bisimilar systems with minimal state space dimension and minimal number of state equations.

In Chapter 5 a possible further question is how to generalize the notion of bisimulation relation to the non-regular matrix pencil.

In Chapter 6 a future research questions can be given in the following. The first question is to solve the disturbance decoupling problem for linear complementarity systems. The second open question is to study the disturbance decoupling problem where we also consider reset rules in the dynamics of linear systems with complementarity switching.

In Chapter 7 there are several research questions that are still open. These can be summarized as follows. The first question is to find a formal algorithm to construct a controller achieving a given specification system.The second question is to derive necessary and sufficient conditions for the existence of a feedback controller. The third question is the generalization to differential-algebraic systems.